Is it a Problem that Physics is Mathematical?

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Physicalism – the view that fundamental reality is entirely physical – is a very popular view. Whilst there is significant opposition to it, this opposition tends to focus on challenging the potential of physicalism to account for consciousness. Susan Schneider, in her timely paper 'The problem of the physical base' hones in on another source of metaphysical difficulty for physicalism: its reliance on mathematics to tell the basic story of the universe. Galileo's declaration that mathematics is the language in which the great book of the universe is written has been of great *practical* benefit, allowing us to build mathematical models that predict the behaviour of matter with great accuracy, and leading ultimately to marvellous technology. But this pragmatic leap forward arguably brings with it profound difficulties for those interested in metaphysics: in finding out what reality is really like. Can we make sense of a purely mathematical universe, and even if we can does the resulting view live up to the expectations many philosophers invest in physicalism?

This is a much neglected problem, and Schneider's paper is to be admired not only for drawing our attention to it, but also for providing insightful analysis of various dimensions of it. I have, however, some disagreement in detail with how we are to understand the problem, which I will try to outline in what follows.

(i) Who's afraid of abstraction?

Schneider's central contention is that the involvement of mathematics in the physicalist's characterisation of fundamental reality entails that physical reality is, at least in part, abstract.

> The physicalist holds that everything is physical, being either a fundamental physical entity or depending upon fundamental physical entities in the base. Now, in the face of the abstract nature of physics, one could raise the following worry. It seems that the fundamental properties and particulars in the physicalist's base – the building blocks of the physicalist's ontological universe – are *individuated* mathematically. That is, mathematical entities seems to make the objects and properties figuring in the physical base the types of objects and properties they are. And if something individuates something else it is generally considered to be part, or even all, of the entity's nature, unless an argument is provided to the contrary….So…why is the physical base not abstract, at least in part, if fundamental physical entities seem to be individuated by abstracta?

Let us take a step back. The mere fact that an entity we think of as concrete has mathematical properties does not seem to me to threaten its claim to be concrete. Suppose I'm having two pains simultaneously: one in my left arm and one in my right arm. And suppose the two pains are qualitatively very similar, except that the one in my right arm is more intense than the one in my left arm. We can truly say that the right arm pain is *greater than* the left arm pain, along a certain dimension of phenomenality. Now *being greater than* would seem to be a mathematical property, but the fact that my consciousness instantiates this mathematical property does not take away from

the claim of either my consciousness or its bearer to be a fully concrete entity. After all, the fact that my consciousness instantiates this property is wholly grounded in its concrete phenomenal character: the mere fact that my pains have the phenomenal character they do entails that they bear this mathematical relationship to each other. The mathematical property instance is grounded in the concrete property instance rather than vice versa.

This mathematical property is a contingent feature of my conscious experience. But even if a putative concrete entity *E* has a mathematical property as part of its essence, I submit that this does not in itself threaten *E*'s claim to be a concrete entity. Suppose Plato was right that the soul has three essential aspects: reason, appetite and spirit. It would follow that a mathematical concept – the concept of threeness – is essential for characterising the essence of the soul. But I can't see why this fact would make the soul partly abstract. Imagine God drawing up the blueprints for the soul: She initially has one component, then She adds another, and finally She decides to add one more for good measure. In adding each of the latter two components God adds nothing abstract to the soul; She merely makes plans for more concrete reality. How could adding more concrete reality make reality more abstract?

These cases, in my view, pose no threat to the concrete-ness of reality because the mathematical properties in question are realised by concrete properties. The trouble comes when it is proposed that physical entities have *nothing but* mathematical properties. If the fundamental nature of physical reality is entirely comprised of abstract mathematical properties it surely follows that physical reality is, in its fundamental nature, abstract.

Even then I don't think the view can be dismissed too quickly. The physicalist may claim that reality is both abstract and concrete, by holding that concrete reality can be reduced to, or is identical with, abstract reality.[1] This may sound like a contradiction at first, but then so does the physicalist proposal to identify of the mental with the physical, or for that matter the idealist's proposal to identify the physical with the mental. In each case, the opponent of the reduction owes the reductionist an argument as to why the proposed reduction is not coherent.

Schneider worries about how such a view would account for space and time, and causal relations, given that abstracta are 'by definition immutable, acausal, and outside of space and time.' But the reductionist can allow that space, time and causality are real, whilst at the same time denying that they exist at the fundamental level (just as the physicalist accepts that mentality is real whilst denying that it exists at the fundamental level). As Schneider herself comments, many speculative theories of fundamental reality claim that spacetime is an emergent entity. And Humean theories promise to reduce causal facts to facts about regularities.

In my view, the biggest challenge to the reduction of the abstract to the concrete is 'Newman's problem', the difficulty raised by Max Newman in response to Russell's 1928 *Analysis of Matter*.[2] The essence of the problem is that any purely formal description of the physical world is near vacuous. Here is an illustrative example due to Peter Ainsworth:

> For example, being told that a system has domain D = {a, b, c} (where a, b, and c are arbitrary names for three distinct but unspecified objects) and instantiates a relation R = {⟨a, b⟩, ⟨a, c⟩, ⟨b, c⟩} tells us no more than that the system consists of three objects, because some elementary set-theory reveals that any three objects instantiate seven non-empty one-place relations, 511 non-empty two-place relations (of which R is one) and 134,217,727

[1] As defended, for example, by Ladyman et al 2007

[2] Newman 1928.

> non-empty three-place relations. Being told that they instantiate R is both trivial (insofar as it follows from some elementary set-theory) and perversely specific (insofar as R is just one of the 134,218,245 non-empty relations they instantiate). Thus being told that the system has structure <D, R> is being told no more than that it contains three objects, because any system containing three objects can be taken to have this structure, along with a vast number of other structures (any tuple whose first member is D and whose other members are amongst the 134,218,245 relations instantiated by the members of D is a structure that can be taken to be possessed by any system containing three objects).[3]

The existence of a huge number of set-theoretic relations follows necessarily from the mere existence of the right number of objects, and any purely formal description of reality can be interpreted in terms of such set-theoretic relations. The upshot is that a purely formal description of reality can tell us no more than how many objects there are: if the right number of objects exist, enough set-theoretical relations are entailed to give the description an interpretation. Surely our knowledge of the physical universe is not this thin?

We can avoid Newman's problem only if our theory of the world contains at least one non-formal (i.e. non-logico-mathematical) notion. Fortunately for the physicalist, physics does not seem to be entirely mathematical; it seems to contain causal or at least nomic notions. It is controversial whether fundamental physics actually involves causal notions, but at the very least it involves non-mathematical nomic notions such as that of a *law of nature*. If we allow that physical theory specifies relations of natural necessity, then we add at least one concrete element to physics, and hence potentially (more would need to be said) avoid Newman's problem.

Adopting this strategy leads the physicalist to a form of causal structuralism: the view that the nature of fundamental properties is given in purely causal terms. However, this is to leap out of the frying pan and into the fire, as there are powerful arguments against causal structuralism. Most discussed is the worry that causal structuralist attempts to characterise the nature of matter lead either to a vicious regress or a vicious circle.[4] According to causal structuralism, we understand the nature of a disposition only when we know the behaviour it gives rise to when it's manifested. For example, the manifestation of flammability is burning; we only know what flammability is when we know that it's manifested through burning. The problem is: assuming causal structuralism, the manifestation of any disposition will be another disposition, and the manifestation of that disposition will be another disposition, and so on ad infinitum. The buck is continually passed, and hence an adequate understanding of the nature of any property is impossible, even for an omniscient being. In other words, a causal structuralist world is unintelligible.

Let us try to make this clear with an example. According to general relativity, mass and spacetime stand in a relationship of mutual causal interaction: mass curves spacetime, and the curvature of spacetime in turn affects the behaviour of objects with mass (as matter tends, all things being equal, to follow geodesics though spacetime). What is mass? For a causal structuralist, we know what mass is when we know what it does, i.e. when we know the way in which it curves spacetime. But to really understand what this amounts to metaphysically, as opposed to being able merely to make accurate predictions, we need to know what spacetime curvature is. What is spacetime curvature? For a causal structuralist we understand what spacetime curvature is only when we know what it does, which involves understanding how it impacts on objects with mass. But we understand this only when we know what mass is. And so we find ourselves in a classic Catch 22: we can understand the

[3] Ainsworth 2009: 142.

[4] Robinson 1982, Lowe 2006, Goff 2017: ch. 6.

nature of mass only when we know what spacetime curvature is, but we can understand the nature of spacetime curvature only when we know what mass is. G. K. Chesterton said that, 'We cannot all live by taking in each other's washing.' Russell played on this idea in articulating this worry about circularity: 'There are many possible ways of turning some things hitherto regarded as 'real' into mere laws concerning the other things. Obviously there must be a limit to this process, or else all the things in the world will merely be each other's washing.'[5]

What options are left for the physicalist? The only remaining option seems to be some form of *noumenalism*: the view that the nature of the fundamental properties of the world are unknown and perhaps unknowable. How is this view consistent with physicalism? Because the physicalist can claim that the fundamental properties of the world are all and only those *referred* to by the predicates of physics: mass, charge, etc., but that the natures of those properties go beyond what the austere mathematico-nomic vocabulary of physics can ever reveal to us. Physics picks out mass in terms of what it does, but mass itself has an underlying nature about which physics remains silent. Thus physicalism is true in the sense that there are only physical properties (the physicalist may want to add that those properties are wholly non-mental) even though in another sense physics is an incomplete theory of reality – it tells us nothing about the intrinsic natures of the properties it refers to.

At this point – and here after a bit of disagreement with Schneider I find myself returning to agreement with her – physicalism seems to me to have bought its coherence but only at the cost of losing alleged advantage in terms of parsimony over rival theories. Noumenalist physicalists think that fundamentally there are only the properties referred to in fundamental physics; panpsychists and neutral monists can also say this (and if they are Russellian monists they do say this).[6] Panpsychists and neutral monists think that physical science is incomplete, in the sense that it does not reveal the intrinsic natures of physical properties – noumenalist physicalists think this too. If anything panpsychism looks to be more parsimonious than noumenalist physicalism, in supposing that the intrinsic nature of matter is continuous with the intrinsic nature of animate matter.[7]

(ii) The metaphysical foundations of maths

The issues discussed in the previous section seem to me distinct from the question of what the truthmakers are for mathematical truths. To begin to see this, consider that the following two views seem to be compatible:

(A) Fictionalism about the sentences of pure mathematics, e.g. '2=2=4'.

(B) The thesis discussed in the last section, according to which that the essence of physical properties is purely mathematical (let's imagine for the sake of discussion that Newman's problem is somehow avoided).

The vocabulary of physics is mathematical, but the sentences of physics are nonetheless distinct from the sentences of pure mathematics; for one thing the former are contingently true whilst the latter are (if true) necessarily true. It therefore seems consistent to suppose that the mathematically framed sentences of physics, e.g. sentences concerning the contingent existence of quantum wave functions or four-dimensional geometrical structures, have truthmakers, but the sentences of pure

[5] Russell 1927: 325.

[6] For a good collection on Russellian monism, see Alter & Nagasawa 2015. Goff 2017 categorizes and evaluates various forms of Russellian monism.

[7] This is the 'simplicity argument' for panpsychism I defend in more detail in Goff 2017.

mathematics, e.g. '2+2=4', do not. Indeed, this combination of views seems to work quite well together: a proponent can say that almost none of the infinity of mathematical structures quantified over in pure mathematics exist, only those which are quantified over in physics are real.

Schneider thinks that anti-physicalists have certain advantages over physicalists when it comes to offering truthmakers for mathematical truths, but I can't really see this. She claims that physicalist mathematical fictionalists are threatened by vicious circularity: mathematical truths are explained in terms of mental facts (i.e. facts about the fictional representations), mental truths are explained in physical truths, and the physical truths are explained in terms of mathematical truths (completing the circle). But the distinctions made in the last paragraph show, I think, that this isn't quite right. The physicalist mathematical fictionalist is not explaining physical truths in terms of the necessary truths of pure mathematics, but in terms of the mathematically framed contingent truths of physics, truths asserting the contingent existence of various mathematically characterised structures (perhaps wave functions and/or four-dimensional geometrical structures). What the physicalist mathematical fictionalist needs to avoid is quantifying over mathematical entities they don't believe exist, e.g. numbers, but there is no problem with their quantifying over abstract entities they do take to exist, e.g. quantum wave functions.

Moreover, I don't think this is quite the right way to think about fictionalism. Mathematical fictionalists do not explain mathematical truths in terms of more fundamental truths; rather they deny that they're true at all. In principle, for the fictionalist, reality can be exhaustively described without appealing to the truths of pure mathematics. Schneider rightly points out that avoiding quantifying over the entities of pure mathematics is going to be especially difficult if spacetime turns out to be an emergent entity, as some quantum gravity theories suggest, but this is a problem faced by non-physicalist as well as physicalist mathematical fictionalists.

Perhaps anti-physicalists who take mental facts to be fundamental have a bit more leeway, in that they may account for the truths of pure mathematics in terms of mental facts, and then freely quantify over the entities of pure mathematics, e.g. numbers, in their physics. But such views seem to me rather implausible (at least if the relevant mental truths are contingent rather than necessary), for the simple reason that the truths of pure mathematics are necessary. If the truthmaker for '2+2=4' is some contingent mental fact M, then there will be some possible world, namely that in which M does not exist, in which '2+2=4' is false. But this is absurd as '2+2=4' is true in all possible worlds.[8]

In my view this is the biggest challenge for the physicalist who wants to be a realist about pure mathematics. If all that exists are *contingent* physical facts, then there doesn't seem to be anything that could possibly ground the *necessary* truths of mathematics. Perhaps mathematical truths can be dispensed with (hard to do!), but even then we are left with the necessary truths of logic and no necessarily existent entities to ground them. The kind of conventionalism about logical truths popular among the logical positivists is now broadly discredited, not least because it's hard to see how contingent conventions of human beings could ground a necessary truth such as the law of non-contradiction.[9]

[8] This needs to be slightly qualified. It could be that in the possible world in which M does not exist, there exists some other truthmaker for '2+2=4'. Nonetheless, it's hard to see how an account of mathematical truth in terms of contingently mentality could provide truthmakers (for the truths of pure mathematics) in all possible worlds.

[9] See, for example, Sider 2012: 6.5.

Of course some physicalists restrict their physicalism to the concrete world, and are happy to adopt some form of Platonism about abstract objects. But for the purer form of physicalism according to which all that it exists is the contingent physical world, some form of anti-realism (fictionalism or expressivism) about the necessary truths of logic and mathematics seems to be inevitable. Thus, I think there is a deep difficulty for (the purer sort of) physicalism arising from the need to account for the truths of mathematics, but again my understanding of the problem is a little different from Schenider's.

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